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EDDY CURRENT ANALYSIS FOR 3-D PROBLEMS USING THE BOUNDARY ELEMENT METHOD

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INTRODUCTION

A modified boundary integral equation/boundary element method (BIE/BEM) is being developed for eddy current problems in three dimensions. Maxwell's equations governing the eddy current problems are formulated in two sets of BIE's, one for the electric field and the other for the magnetic field. These BIE's involve both the field and the normal derivative of the field, for both exterior (air) and interior (metal) regions. In addition to the usual set of interface conditions involving only the field, a set of interface conditions involving the normal derivatives of the field is derived by applying Maxwell's equations near the interface. The present approach represents a departure from the existing BIE formulation for eddy current problems (see, e.g. [1-3]) in which normal derivatives of the field do not explicitly appear. However, the approach here draws heavily on the authors' experience and success with and code development for ultrasonic scattering

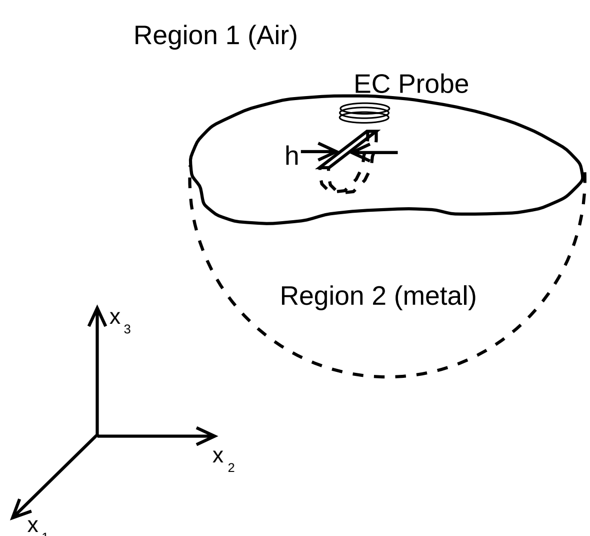

Fig. 1. A surface open crack (notch).

especially from cracks. Singular integrals in all BIE's are transformed into weakly singular ones and therefore no special integration schemes are required. Conforming quadratic and non-conforming quadratic boundary elements are implemented in this study for the discretizations of the BIE's. Preliminary numerical results from simple geometries (sphere and half space) indicate the accuracy and efficiency of the developed solution procedure. A study of surface cracks is underway where the conventional BIE and hypersingular BIE are combined to examine cracks with a variety of characteristics, e.g. tightly closed cracks, open cracks and rough cracks. Finally, it is pointed out that the BIE formulations developed here for eddy current problems can be readily applied to problems of eletromagnetic scattering from arbitrary 3-D bodies.

BOUNDARY INTEGRAL EQUATIONS (BIE'S) FOR ELECTRIC FIELD

Since each component E_i of the electric field satisfies the Helmholtz equation under the divergence free condition, we can employ the following conventional BIE (CBIE)

$$C(P_o)E_i(P_o) + \int_S \frac{\partial G(P,P_o)}{\partial n} E_i(P)dS(P) = \int_S G(P,P_o)\frac{\partial E_i(P)}{\partial n}dS(P) + \int_V \alpha\, G(Q,P_o)\Im_i(Q)dV(Q) \qquad (1)$$

e.g., for the exterior domain (air), where S is the (air/metal) interface (see Fig. 1), n the normal, V the volume of the coil (probe), C and α are constants. The Green's function

$$G(P,P_o) = \frac{1}{4\pi r}e^{ikr}$$

with r being the distance between the field point P and the source point P_o, and k the wave number. Note that the normal derivative of the E field appears explicitly as in the BIE formulations for acoustic problems. The first surface integral in Eq.(1) contains a strong singularity and thus a weakly singular version of Eq.(1) is used in the actual numerical implementation. A similar CBIE, without the coil term, is applied for the interior domain (metal).

For crack or crack-like problems, the CBIE's, like Eq.(1), will degenerate when they are collocated on both sides of the crack surface, see Fig.2 (a). This degeneracy is

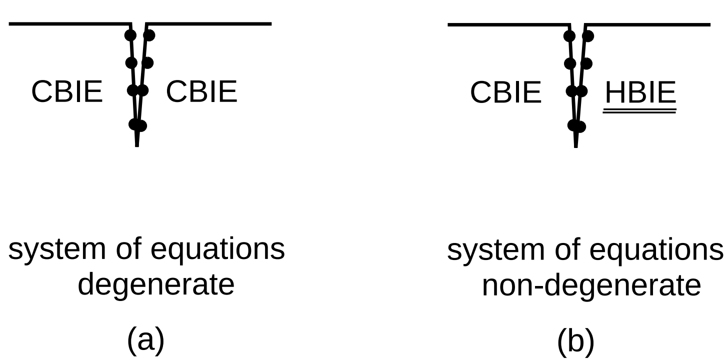

Fig. 2. CBIE and HBIE for crack problems.

indicated numerically by a large condition number of the system of equations generated from CBIE's. Detailed discussions can be found in [4]. One remedy for this degeneracy is to employ the hypersingular BIE's (HBIE's) obtained by taking (normal) derivatives of the CBIE's. Degeneracy will also occur if HBIE's are applied alone to crack problems. However, a combination of the CBIE and HBIE, e.g., collocating the CBIE on one surface of the crack and the HBIE on the other, Fig.2 (b), can furnish a non-degenerate system of equations, no matter how close the two crack surfaces are [4].

The hypersingular BIE derived by taking the normal derivative of Eq.(1) can be written as

$$\tilde{C}(P_o)\frac{\partial E_i(P_o)}{\partial n_o} + \int_S \frac{\partial^2 G(P,P_o)}{\partial n \partial n_o} E_i(P)dS(P) = \int_S \frac{\partial G(P,P_o)}{\partial n_o}\frac{\partial E_i(P)}{\partial n} dS(P) + \int_V \alpha \frac{\partial G(Q,P_o)}{\partial n_o} \Im_i(Q)dV(Q) \qquad (2)$$

for the exterior domain (air). The integral on the left hand side contains a hypersingularity $O(1/r^3)$ for 3-D problems. Interpretations of and computation schemes for strongly singular ($O(1/r^2)$) integrals fail. Nevertheless, with recent development of research on HBIE's, effective computation with HBIE's is possible and essential for the present problems. Here, we employ the following weakly singular form of Eq.(2)

$$\begin{aligned}
&\frac{\partial E_i(P_o)}{\partial n_o} + \int_S \frac{\partial^2 \bar{G}(P,P_o)}{\partial n \partial n_o}\left[E_i(P) - E_i(P_o) - E_{i,k}(P_o)(x_k - x_{ok})\right] dS(P) \\
&\quad + \int_S \frac{\partial^2}{\partial n \partial n_o}\left[G(P,P_o) - \bar{G}(P,P_o)\right] E_i(P)dS(P) \\
&= \int_S \frac{\partial \bar{G}(P,P_o)}{\partial n_o}\left[E_{i,k}(P) - E_{i,k}(P_o)\right] n_k(P)dS(P) \\
&\quad + \int_S \frac{\partial}{\partial n_o}\left[G(P,P_o) - \bar{G}(P,P_o)\right]\frac{\partial E_i(P)}{\partial n} dS(P) \\
&\quad + \int_V \alpha \frac{\partial G(Q,P_o)}{\partial n_o} \Im_i(Q)dV(Q), \qquad \forall P_o \in S
\end{aligned} \qquad (3)$$

which was originally developed and has been used successfully for acoustic wave problems [5] to remove the fictitious eigenfrequencies in the BIE formulations. All integrals in Eq.(3) are at most weakly singular and hence regular integration quadrature is sufficient to compute these integrals. It is also easier to implement higher order boundary elements with Eq.(3). The discretization of Eq.(3) is straightforward and the CPU time for setting up the system of equations for Eq.(3) is only slightly longer than that for Eq.(1).

INTERFACE CONDITIONS

At each point (node) on the interface (boundary), there are six scalar variables (three components of the *E* field and three components of the normal derivative of the *E* field) for the exterior domain and another six similar variables for the interior domain. Altogether, there are twelve unknown variables for the present BIE formulation. On the other hand, three scalar equations can be generated at each node for the exterior domain and another three equations for the interior domain, using either CBIE (1) or HBIE (3). We have only six equations available. Thus, in order to apply the present BIE formulation, which involves explicitly the normal derivative of the E field, we need six interface conditions.

Let E_k be the k-th *covariant* component of E vector in the local $\xi^1\xi^2\xi^3$ system (established, e.g., on a boundary element) with ξ^1 and ξ^2 in tangential directions and ξ^3 in the normal direction. The traditional interface conditions are (the superscripts 1 and 2 refer to exterior domain (air) and interior domain (metal), respectively)

$$E_1^{(1)} = E_1^{(2)}, \qquad E_2^{(1)} = E_2^{(2)}, \qquad \beta^{(1)}E_3^{(1)} = \beta^{(2)}E_3^{(2)}, \tag{4}$$

and the following additional conditions can be derived by applying the field equations near the interface

$$\begin{aligned}
\frac{\partial E_1^{(1)}}{\partial \xi^3} - \frac{\alpha^{(1)}}{\alpha^{(2)}}\frac{\partial E_1^{(2)}}{\partial \xi^3} &= \left[1 - \frac{\alpha^{(1)}\beta^{(1)}}{\alpha^{(2)}\beta^{(2)}}\right]\frac{\partial E_3^{(1)}}{\partial \xi^1}, \\
\frac{\partial E_2^{(1)}}{\partial \xi^3} - \frac{\alpha^{(1)}}{\alpha^{(2)}}\frac{\partial E_2^{(2)}}{\partial \xi^3} &= \left[1 - \frac{\alpha^{(1)}\beta^{(1)}}{\alpha^{(2)}\beta^{(2)}}\right]\frac{\partial E_3^{(1)}}{\partial \xi^2}, \\
\frac{\partial E_3^{(1)}}{\partial \xi^3} - \frac{\partial E_3^{(2)}}{\partial \xi^3} &= \frac{1}{2g}\frac{\partial g}{\partial \xi^3}\left[\frac{\beta^{(1)}}{\beta^{(2)}} - 1\right]E_3^{(1)},
\end{aligned} \tag{5}$$

where $\alpha = i\omega\mu$, $\beta = -i\omega\varepsilon$ or σ, and g is the determinant of the metric tensor.

SOLUTION STRATEGY

After imposing the above six interface conditions at each node to eliminate, say, the variables for the interior domain, we can write the following system of equations for the discretized BIE's for exterior and interior domains

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{E} \\ \dfrac{\partial \boldsymbol{E}}{\partial n} \end{Bmatrix} = \begin{Bmatrix} b \\ 0 \end{Bmatrix} \tag{6}$$

where $\boldsymbol{E}$ and $\partial\boldsymbol{E}/\partial n$ are vectors composed of E_i and $\partial E_i/\partial n$ at all the nodes. By solving the above system, we obtain the E field and its normal derivative on the interface.

The magnetic field can be obtained using two approaches. One approach is to derive the H field from the resulting E field and its normal derivative by employing the Maxwell equation on each element. This is a quick way to obtain the H field, but some accuracy might be lost if any numerical differentiation is applied. The other approach is to solve a different set of BIE's derived for the H field. This approach may result in the H field of equivalent accuracy as compared to the E field, but it demands extra CPU time.

Two types of boundary elements are applied for this study, namely, the conforming quadratic elements and the non-conforming quadratic elements (see [5]).

A conducting sphere in a uniform, time harmonic magnetic field, for which the exact solution is available, was studied first, in order to check the BIE formulation and the interface conditions. The distribution of the *E* field over the surface of the sphere (radius = 1 cm) is plotted in Fig. 3. Comparable (or better) numerical results as compared with those reported in [3], where 80 constant elements were used for the whole sphere, were obtained here with only 12 (or 16) conforming quadratic elements on the whole sphere.

The eddy current in a half space was then studied and the the results are shown in the Fig. 4 - Fig. 7. The radius of the single coil probe a=1 mm and lift off d=1 mm. A truncated region (with radius $R = 10a$) of the surface of the half space was modeled with elements and contributions from integrals on the surface at infinity are taken into account analytically in the BIE formulation. Fig. 4 and Fig. 5 are results using conforming elements at two different frequencies. It is noticed that at the higher frequency (Fig. 5, f=500 kHz), more elements are needed in order to achieve accurate results, which is typical in the BEM practice for frequency dependent problems. Fig. 6 and Fig. 7 are comparisons of the CBIE and the HBIE, using non-conforming elements, at two frequencies. It is shown that the accuracy of the CBIE and the HBIE are about the same.

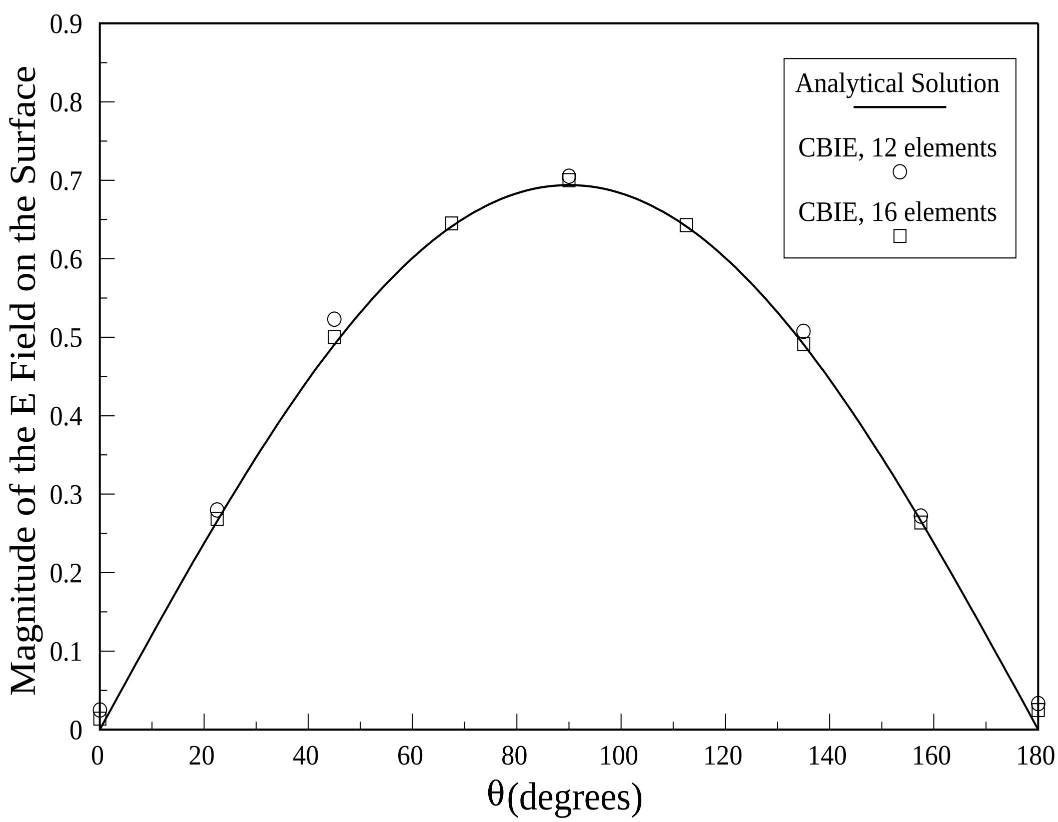

Fig. 3. Results for the sphere using conforming elements, f = 500 Hz.

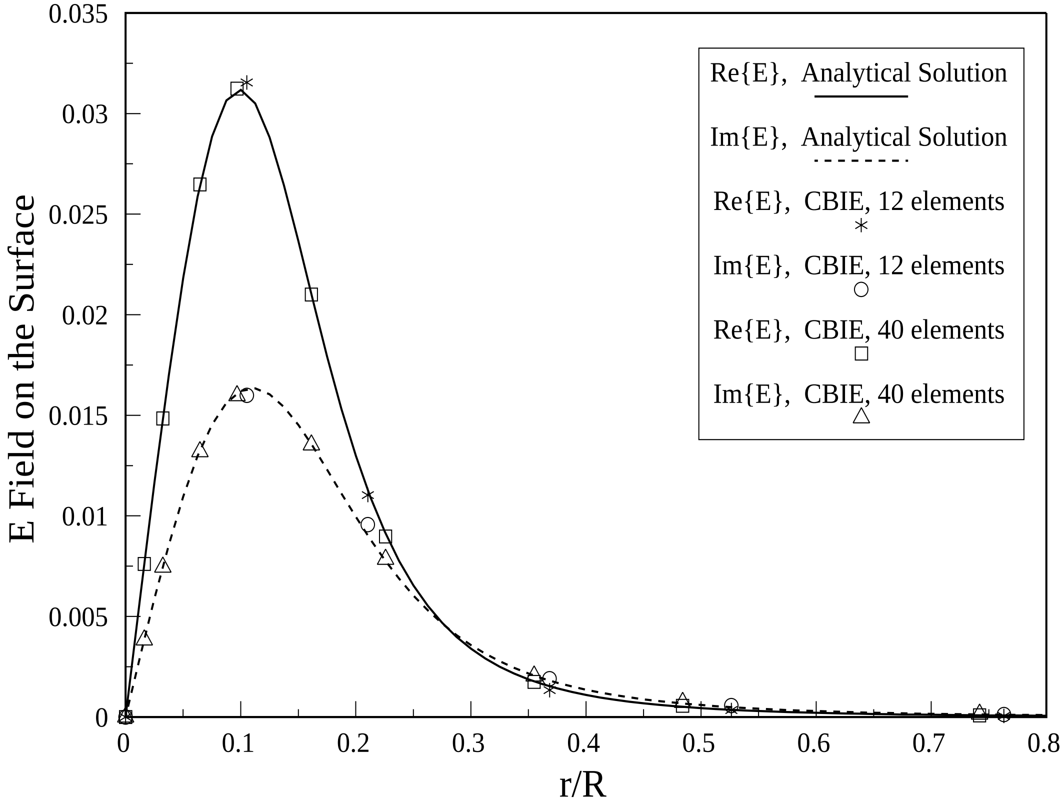

Fig. 4. Results using conforming elements, f = 50 kHz.

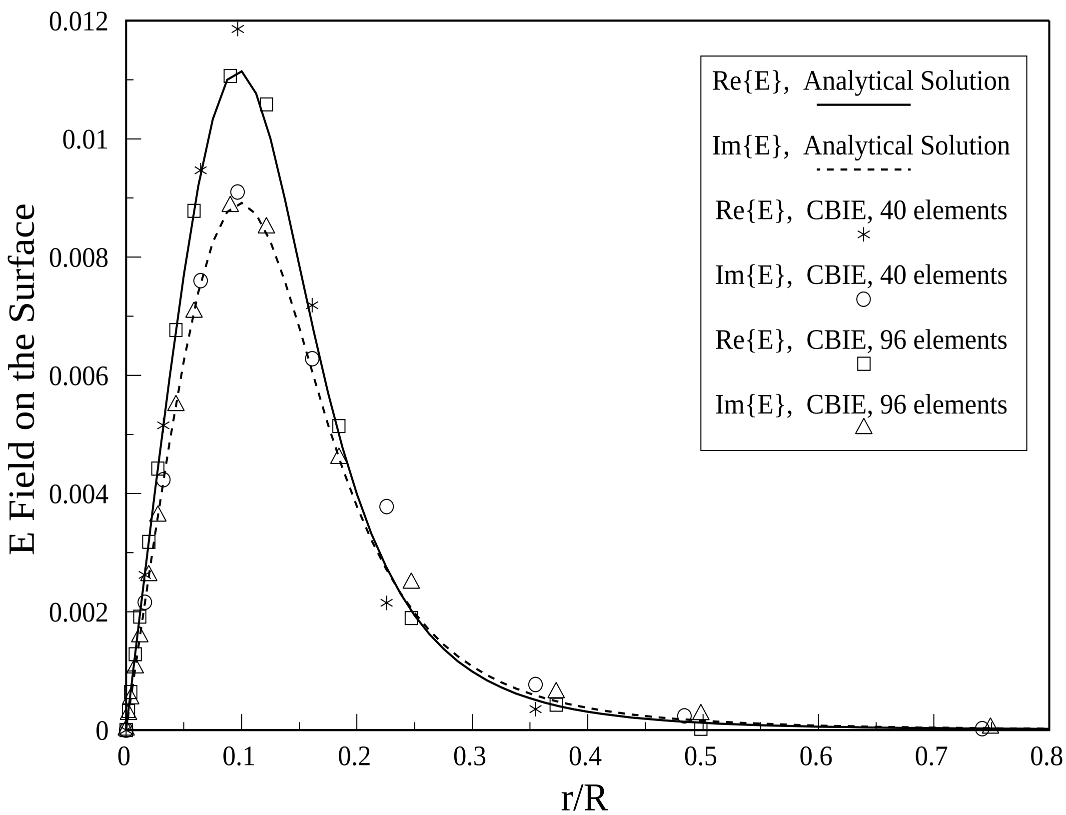

Fig. 5. Results using conforming elements, f = 500 kHz.

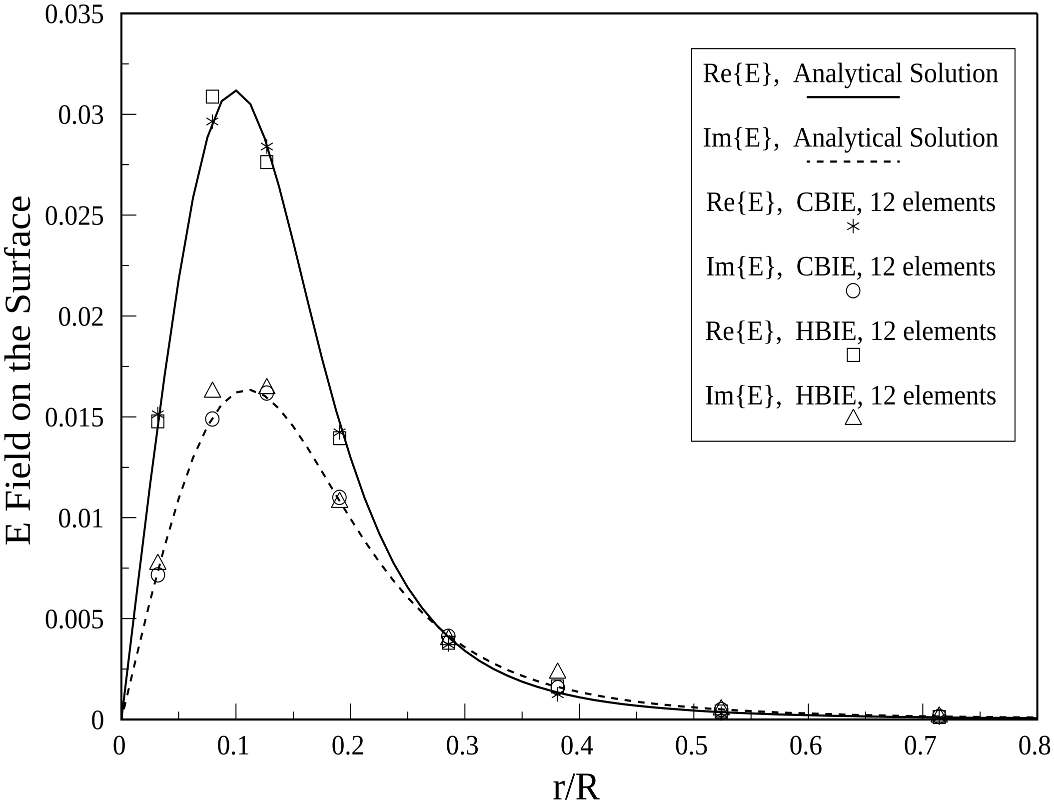

Fig. 6. Comparison of CBIE and HBIE, non-conforming elements, f = 50 kHz.

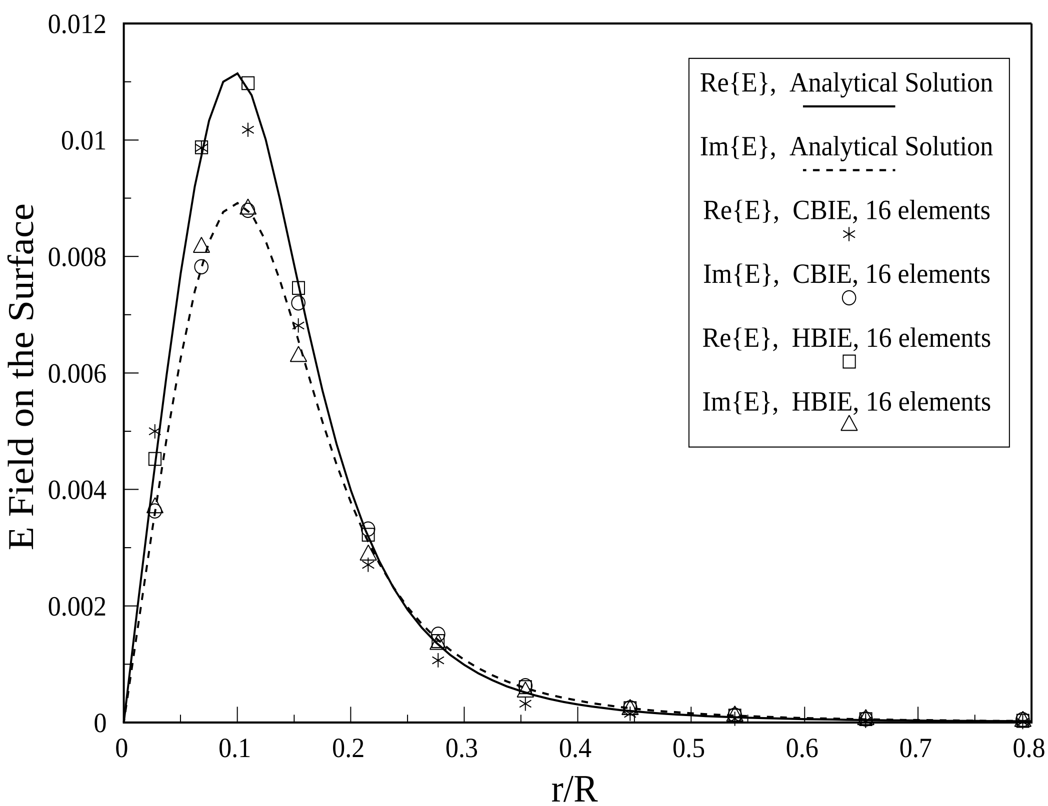

Fig. 7. Comparison of CBIE and HBIE, non-conforming elements, f = 500 kHz.

DISCUSSION

The basic effectiveness and accuracy of the developed BIE formulation (using CBIE or HBIE) and solution scheme are indicated by the above numerical examples. A study of surface cracks is underway where the CBIE and the HBIE are combined to form a non-degenerate BIE formulation for a variety of cracks as mentioned earlier.

Of particular interest is the open crack configuration shown in Fig. 1, with "opening parameter" h, wherein the eddy current field as a function of h is the focus. Most experimental calibrations are done for $h \neq 0$ whereas most computations have been limited to the tight crack $h=0$. Now that a non-degenerate formulation for arbitrary small h is available, new confidence intervals can be established for the calibrations.

This work is being extended to cracks which intersect surfaces at corners and grooves using probes with a ferrite core. Overall, the mathematics of the BIE formulas are such that they are applicable to electromagnetic scattering from thin, cracklike shapes in the vicinity of curved and possibly rough surfaces. Finally, acoustic and ultrasonic scattering problems, which may be regarded as the mathematical counterparts of the present problem, are also under investigation.

ACKNOWLEDGMENTS

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