A Higher-Order FEM for Vibration Control of Composite Plates with Distributed Piezoelectric Sensors and Actuators

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Summary

The active vibration control of laminated composite plates using distributed piezoelectric sensors and actuators (Fig. 1) is studied numerically with a higher-order finite element method (FEM). Reddy's simplified higher-order theory for transverse shear deformation, which is found to be much more accurate for both thin and thick composite plates as compared with other theories, is employed for the finite element models of the composite plates and piezoelectric devices. On the other hand, the finite elements are based on higher-order Lagrange interpolations in order to provide higher accuracy and efficiency in the FEM process. The analysis of the system of composite plates with piezoelectric sensors and actuators is formulated in such a way that the actuating effects are introduced into the dynamic equations through the damping coefficient matrix. The vibration control of a rectangular composite plate with the piezoelectric devices is studied to validate the developed FEM approach. The minimal number and optimal locations of the piezoelectric sensors and actuators are investigated intensively through ten cases. The numerical results show that by using the piezoelectric devices, the dynamic response of the plate can reduce as much as 40% and also decay at a faster rate. It is found that the best location for the piezoelectric actuators is in the high strain areas of the structure, and that, in this case, more smaller, segmented piezoelectric actuators can have better control effects than a few larger, continuous ones.

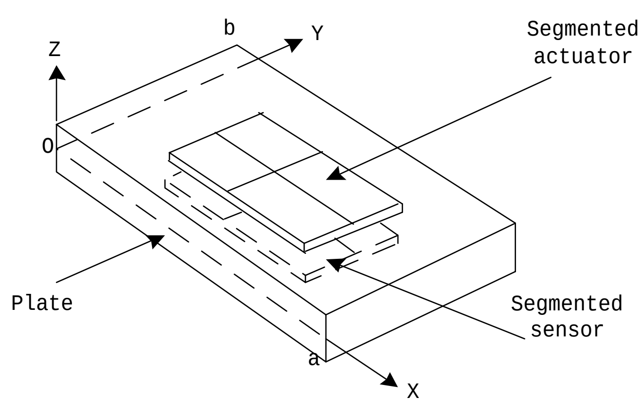


Figure 1. Active vibration control of a composite plate with piezoelectric sensors and actuators.

Overview of the Finite Element Formulation

The finite element method has been applied to the vibration analysis of piezoelectric solids for quite some time since the work by Allik and Hughes using 3-D elements [1]. For the dynamic analysis of plates and shells, it is in general more efficient to employ 2-D finite elements based on plate and shell theories, since the piezoelectric sensors and actuators bonded on the surfaces of the plates or shells are in general very thin. Some of the recent work in this area can be found in [2-4] for the vibration control or measurement of thin, isotropic plates and shells with distributed piezoelectric sensors and actuators covering the whole upper and lower surfaces; in [5, 6] for the same topics but with segmented piezoelectric films covering only part of the surfaces; and in [7-9] for the vibration control analysis of anisotropic piezoelectric laminated plates. Composite plates with distributed piezoelectric sensors and actuators was studied recently in [10] with eight-node quadratic elements. In Ref. [10], the composite plate element is based on Reddy's higher-order theory for laminated composite plates [11, 12]. However, the distributed piezoelectric sensors and actuators modeled in [10] cover the whole top and bottom surfaces of the composite plate, which may not be the most efficient way to employ the piezoelectric elements on composite plates, as shown in the current study.

In the current study, a Lagrange type of element, which can provide arbitrarily higher-order polynomials in the shape functions, is developed for the composite plates with distributed piezoelectric sensors and actuators. These piezoelectric devices are segmented and bonded on parts of the top and bottom surfaces of the composite plate, which provides the flexibility in studying the optimal locations of these piezoelectric devices in the active vibration control.

Reddy's simplified higher-order theory for laminated composite plates [11, 12] is found during this study to be more accurate and efficient for the vibration analysis of both thin and thick composite plates, compared to the models based on Kirchhoff thin plate, Mindlin-Reissner thick plate and other higher-order shear deformation (such as LCW) theories. In Reddy's simplified higher-order theory, the displacement field is represented by

$$\begin{aligned} u(x,y,z,t) &= u_o(x,y,t) + z\varphi_x(x,y,t) + z^3\phi_x(x,y,t), \\ v(x,y,z,t) &= v_o(x,y,t) + z\varphi_y(x,y,t) + z^3\phi_y(x,y,t), \\ w(x,y,z,t) &= w_o(x,y,t), \end{aligned} \tag{1}$$

where u_o, v_o and w_o are the displacements of a point (x, y) on the midplane; φ_x and φ_y are the rotations of the normal to the midplane about the y and x axes, respectively; ϕ_x and ϕ_y are higher-order terms to be determined [11, 12]. In this study, the inplane displacements are not considered ($u_o = v_o = 0$).

At a typical node i of a finite element for the composite plate, the generalized displacement vector is defined as $\mathbf{d}_i(t) = \{w_o, \ \ \varphi_x, \ \ \varphi_y, \ \ \phi_x, \ \ \phi_y\}^T$. The generalized displacement vector at a point inside the element is given by

$$\mathbf{d}(t) = \sum_{i=1}^{N_e} N_i(\xi,\eta)\mathbf{d}_i(t), \tag{2}$$

in which N_e is the number of nodes on the element and N_i the shape function in local coordinates (ξ,η) for node i. In this study, the following Lagrange interpolation functions are employed as the shape functions:

$$N_i(\xi,\eta) = \left(\prod_{j \neq i}^{m} \frac{(\xi - \xi_j)}{(\xi_i - \xi_j)} \right) \left(\prod_{k \neq i}^{n} \frac{(\eta - \eta_k)}{(\eta_i - \eta_k)} \right), \tag{3}$$

where m and n are the numbers of nodes in the ξ and η directions, respectively. The advantages of using the Lagrange interpolations are the flexibility of increasing the order of polynomials and the high accuracy in the analysis of composite plates, despite the disadvantage that some internal nodes are needed on these elements.

Based on the assumptions in (1) and the interpolations (2) and (3), the finite element equation for the dynamic analysis of a composite plate can be written as:

$$[M]\left\{\ddot{d}^e\right\} + [C_c]\left\{\dot{d}^e\right\} + [K]\{d^e\} = \{F_c^e\}, \tag{4}$$

following the standard FEM procedure for dynamic problems, where $\{d^e\}$, $\left\{\dot{d}^e\right\}$ and $\left\{\ddot{d}^e\right\}$ are the nodal elastic displacement, velocity and acceleration vectors, respectively; $[M]$, $[C_c]$ and $[K]$ the mass, damping and stiffness matrices, respectively; and $\{F_c^e\}$ the forcing vector.

When the piezoelectric elements are applied on the top and bottom of the composite plate as actuators and sensors, respectively, the FE equation can be shown to have the following form:

$$[M]\left\{\ddot{d}^e\right\} + \left([C_c] + [\overline{C}]\right)\left\{\dot{d}^e\right\} + [K]\{d^e\} = \{F_c^e\}, \tag{5}$$

where $[\overline{C}] = [K_{ua}]GG_c[K_e^s]^T$ with $[K_{ua}]$ and $[K_e^s]$ being two stiffness-like matrices associated with the elastic-electric coupling, and G and G_c are the gains of the amplifier (controller) and the charge amplifier, respectively, for the actuator in the feedback control. Derivations of Eq. (5) is rather lengthy and the details, as well as the control mechanism, can be found in the work [3, 4, 10].

From Eq. (5), one can conclude that the forces acting on the composite plate by actuators are equivalent to the effect of additional damping on the structure. The new damping matrix is composed of two parts, one from the original structural damping and one from the feedback control forces of the actuators. The application of the actuators can effectively increase the damping of the new system and thus suppress the vibration of the structure.

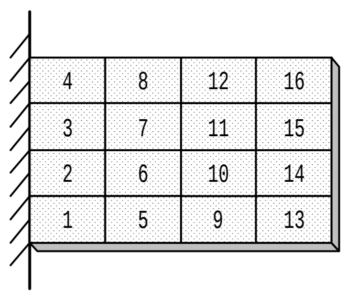
Numerical Examples

The developed higher-order Lagrange elements are first tested on a composite plate, as shown in Fig. 2 (without the piezoelectric elements), to compute the natural frequencies of the plate. In this case, the results can be validated using other methods. The Lagrange plate elements based on Reddy's theory were found to be extremely accurate for composite plates with both small and large thicknesses (e.g., using only one element with 7×7 nodes to model the whole plate).

To verify the developed finite element formulation and the control mechanism for active vibration control, the rectangular clamped composite (unidirectional glass-epoxy) plate with piezoelectric (PVDF) sensors and actuators, as shown in Fig. 2, is analyzed. The plate is discretized using 16 nine-node (3×3) Lagrange elements and a shaded element indicates that the piezoelectric sensor and actuator are applied on the top and bottom surfaces of the plate on that element location.

The transient response of the plate due to an initial velocity specified at the free edge of the plate was analyzed. Ten distribution patterns for the piezoelectric elements were tested and the results for four of these cases (patterns I-IV) are presented in Figs. 3-6. The displacement responses at the upper-right corner node of the plate (on element 16) are plotted for the uncontrolled and controlled conditions (with two different sets of amplifying factor *G* for the feedback control).

Fig. 3 shows that when the piezoelectric elements are placed in the relatively low strain area (middle of the plate), the control effect on the response is insignificant. When the piezoelectric elements are placed near the fixed end of the plate (Figs. 4 and 5), where relatively high strains occur, the control effects improve dramatically. The best or optimal control results in this study (with about 40% reduction in the displacement response) are achieved by placing four piezoelectric elements along the fixed end (Fig. 6). If the piezoelectric elements are applied all over the top and bottom surfaces of the plate, as shown in Fig. 2, the results are almost identical to those in Fig. 6. This suggests that about three quarters of the piezoelectric elements are used ineffectively and thus wasted, if they are applied on the whole surfaces.



Dimensions of the plate:
Length = 200 mm;
Width = 126.5 mm;
Thickness = 5 mm;
Thickness of the piezoelectric films = 20 μm.

Figure 2. The FE model of a composite plate with piezoelectric sensors and actuators.

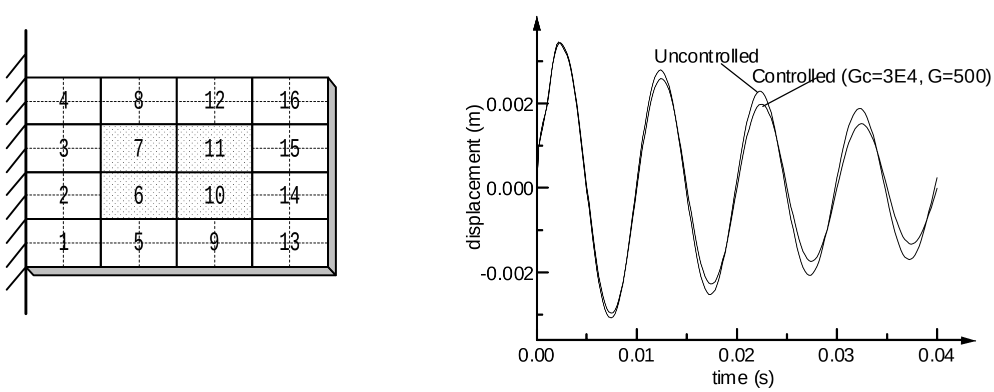


Figure 3. Piezoelectric element distribution pattern I and the displacement response at the corner node.

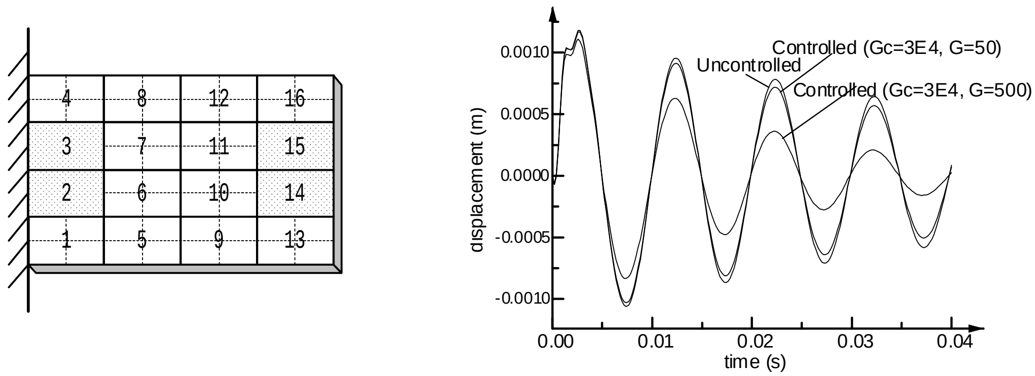


Figure 4. Piezoelectric element distribution pattern II and the displacement response at the corner node.

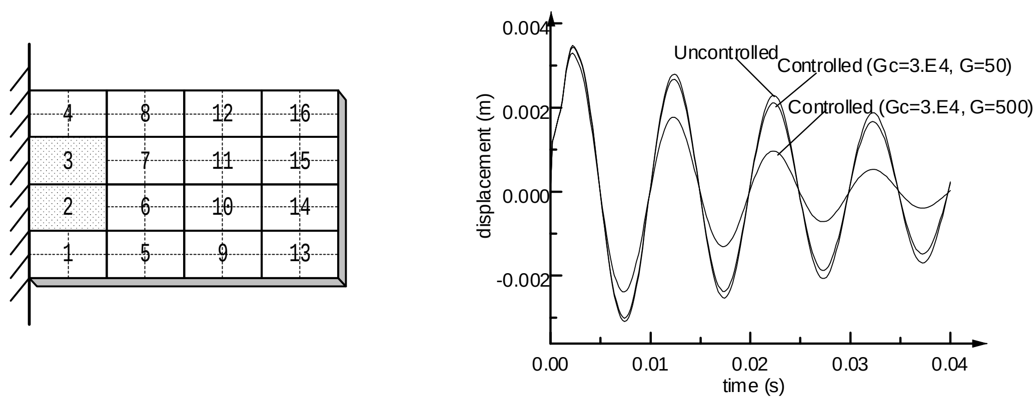


Figure 5. Piezoelectric element distribution pattern III and the displacement response at the corner node.

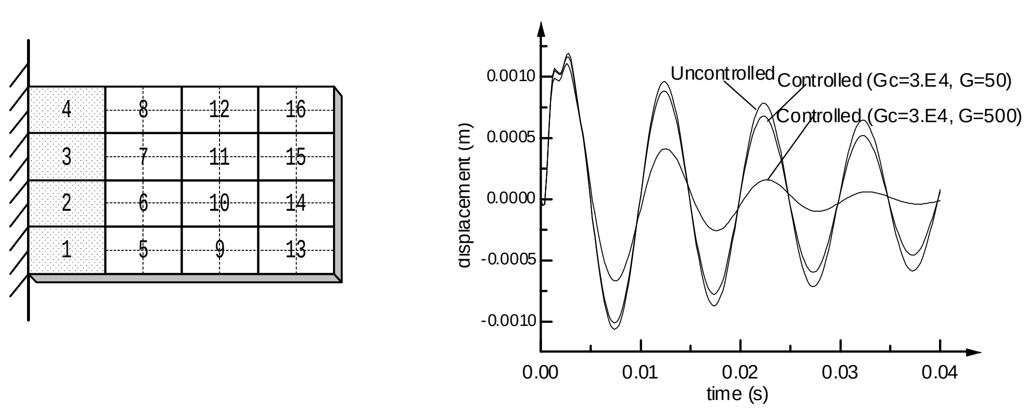


Figure 6. Piezoelectric element distribution pattern IV and the displacement response at the corner node.

Conclusion

The active vibration control of laminated composite plates using the active piezoelectric elements is studied in this paper using the Lagrange type of finite elements based on Reddy's simplified composite plate theory. The study shows that the dynamic response of a composite plate can be effectively suppressed with the application of the piezoelectric elements. Much efficiency and hence savings can be achieved in applying these piezoelectric elements if they are segmented and placed strategically over the high strain areas of the plate, as compared with the option of applying the active elements all over the two main surfaces of the plate. Application of the piezoelectric elements on real structures made of composite plates for active vibration control purposes seems quite promising.

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