A unified boundary element method for the analysis of sound and shell-like structure interactions. I. Formulation and verification

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A unified boundary element method (BEM) is developed in this paper to model both the exterior acoustic field and the elastic shell-like structure in a coupled analysis. The conventional boundary integral equation (BIE) for three-dimensional (3D) elastodynamics is applied to thin shell-like structures which can have arbitrary shapes and small thicknesses. The nearly singular integrals existing in the BIE when applied to thin bodies are transformed to nonsingular line integrals and are evaluated accurately and efficiently. For the exterior 3D acoustic domain, the Burton and Miller composite BIE formulation is employed to overcome the fictitious eigenfrequency difficulty (FED) and the thin-shape breakdown (TSB). Conforming C^0 quadratic elements are employed in the discretization of the two sets of BIEs. The developed BIE formulations are valid for both radiation and scattering problems and for all wave numbers. Numerical examples using spherical and cylindrical shells, including nonuniform thickness and nondimensional wave numbers up to 12, clearly demonstrate the effectiveness and accuracy of the developed BEM approach. © *1999 Acoustical Society of America.* [S0001-4966(99)01709-9]

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INTRODUCTION

The effective control of noise and vibration in a structural acoustic system depends largely on the accurate evaluation of the sound–structure interaction which is characterized by the energy transferring back and forth between the acoustic field and the elastic structure. When the structural impedance is comparable to the acoustic impedance, both of the responses of the structure and the sound field can be significantly affected by this sound–structure interaction. Many numerical techniques have been developed for the analysis of the sound–structure interaction problems, since analytical approaches are limited to simple geometries and loading conditions. For a review of the subject on sound and elastic structure interactions, refer to the classical work in Refs. 1–3 and the references therein.

For the numerical analysis of the acoustic wave, the finite element method (FEM),[4] infinite element method (IEM),[5,6] and boundary element method (BEM)[7–15] have been investigated intensively, among others. Detailed reviews and more references for the three major techniques can be found in Refs. 4, 5, and 15. The FEM uses 3D elements to model the 3D acoustic field. When the infinite acoustic field is encountered, the finite-element model has to be truncated at an artificial outer boundary at which an approximate nonreflecting boundary condition is applied. The Sommerfeld radiation condition is in general not satisfied in this early FEM approach. In the early versions of the IEM, a variety of shape functions were used to approximate the spatial decay of the acoustic pressure outside the finite-element model. Recently, a new infinite element approach using a multipole expansion of the acoustic pressure in the field exterior to a spheroid surrounding the structure was developed.[5,6] This multipole expansion in spheroidal coordinates satisfies the Sommerfeld radiation condition automatically at infinity and can converge to the exact solution with only a few layers of 3D acoustic elements outside the structure.

The BEM has long been considered as a rigorous approach to exterior acoustic problems. The Sommerfeld radiation condition is satisfied exactly by the boundary integral equation (BIE) and only the interior boundary (i.e., the outer surface of a structure) needs to be discretized. Therefore, the analysis of structures with simple or complicated geometries, or multiple scatterers, can be performed conveniently by the BEM. Since near field solutions may be sensitive to small features on the surface of the elastic structure, the ability of modeling these small features without additional efforts also makes the BEM attractive. The possible drawbacks in the BEM approach include the nonuniqueness problem which arises when the conventional BIE (CBIE) is applied to an exterior acoustic domain. This problem is also referred to as the fictitious eigenfrequency difficulty (FED), since nonunique solutions arise at the eigenfrequencies of the associated interior problems.[7,8] However, this FED can be circumvented by either the CHIEF method[7] or the Burton and Miller composite formulation.[8] It has been shown in Refs. 12–15 and many others that the Burton and Miller composite BIE formulation, employing a linear combination of the CBIE and the hypersingular BIE (HBIE), is the most effective method to overcome the fictitious eigenfrequency difficulty for exterior acoustic problem and elastodynamic problems.[16] It has also been demonstrated[17–19] that the composite BIE formulation can overcome the thin-shape breakdown (TSB)[18,20] existing in the CBIE when it is applied to domains surrounding

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thin shell-like structures. The hypersingular integral in the composite BIE presents no difficulty at all, since it can be readily transformed into weakly singular integrals and accurately evaluated by ordinary numerical quadrature.[14,15]

For the modeling of elastic structures, the FEM with its various formulations for beams, plates, shells, and solids, is the mostly accepted method in structural acoustic analysis. However, there are many assumptions involved in the beam, plate, or shell elements. Solid elements with proper aspect ratios should be used when high accuracy is demanded. Very large FEM models may result when solid elements are applied to thin shell-like structures. It may also be difficult to generate the FEM mesh for thin structures if the geometry is complicated. On the other hand, the BEM has established itself as a viable alternative or complement to the FEM for both elastostatic and elastodynamic problems (see, e.g., Refs. 16, 19, and 21–29) after its accuracy and efficiency have been demonstrated and the numerical difficulties have been eliminated. For structural acoustic analysis, the BEM based on 3D elastodynamics and using surface discretization is also advantageous, since the proper coupling of the elastic structure and the acoustic field can be ensured using the same surface mesh. However, there have been two major difficulties or concerns when the CBIEs are applied directly to thin bodies (including thin voids or open cracks, thin shell-like structures, and thin layered structures), where two parts of the boundary become close to each other. One difficulty is the possible degeneracy of the CBIE for thin bodies.[18] The other is the difficulty of the nearly singular integrals[18,29,30] which arise when the integration is conducted on a surface with the source point being very close to the surface. Because of these two difficulties, the BEM has been considered unsuitable for thin-body problems for a long time. It has been shown in Refs. 18 and 29 that these two difficulties can be overcome readily with some analytical efforts. The degeneracy, which happens when CBIEs are applied to the two surfaces of a thin void or crack in an exterior-type problem, can be overcome by employing the Burton and Miller composite formulation.[17–19] For an interior-type problem (thin shells, etc.), it has been shown[29] that no degeneracy will happen when CBIEs are applied on both sides of a thin shell. Accurate results for both 2D and 3D thin structures have been obtained after the nearly singular integrals are handled correctly using the line integral approach.[29,31]

For the coupled structural acoustic problem, the most commonly used approach is the FEM/BEM approach,[32–34] which employs the finite elements for the elastic structure and boundary elements for the exterior acoustic field. This approach combines the advantages of both the FEM and BEM. The drawbacks in this approach include the mismatch of the desirable mesh sizes on the interface between the acoustic field and the elastic field. Since the FEM mesh density required for the elastic structure is usually higher than the BEM mesh density for the acoustic field, and a common mesh should be used for the two domains to ensure proper interface conditions, the selected mesh could be unrealistically dense for the acoustic field and the efficiency can suffer. The FEM/FEM (or FEM/IEM) approach, where the structure is modeled by FEM and the exterior acoustic field by the IEM, is regaining popularity in computational structural acoustics, due to recent success.[4–6] The least explored approach is the BEM/BEM approach, which has been studied for bulky elastic structures in Refs. 35 and 36. In Ref. 35, the BEM/BEM approach is applied to the acoustic wave interacting with bulky elastic bodies in the context of the nondestructive evaluation. In Ref. 36, the BEM/BEM approach was tested on both bulky solid and hollow sphere (thick shell with thickness to radius ratio $=0.5$) using isoparametric elements. The effectiveness and efficiency of the BEM/BEM approach were demonstrated clearly with accurate results obtained for both radiation and scattering problems.[35,36] However, the interaction of the acoustic wave with thin shells was not analyzed in Refs. 35 and 36, probably due to concerns of the degeneracy of CBIEs for thin bodies and the difficulty of dealing with nearly singular integrals which were still troublesome to compute a decade ago.

The present paper extends the BEM/BEM approach to the structural acoustic problem for thin shell-like structures which can have arbitrarily small and nonuniform thickness. Two sets of BIEs, one for the elastic structure (finite interior domain) and the other for the acoustic field (infinite exterior domain), are presented. The conventional BIE based on 3D elastodynamics is used for the elastic structure. The nearly singular integrals are transformed into line integrals which are computed very accurately and efficiently, based on the recent development of the BEM for thin structures.[18,19,29–31] The fictitious eigenfrequency difficulty and the thin-shape breakdown in the CBIE are removed by using the composite BIE formulation using a linear combination of the CBIE and HBIE in the acoustic domain. The weakly singular form of the hypersingular BIE[15] is employed, which can be readily evaluated by the usual numerical quadrature. The two sets of BIEs are coupled at the outer (wet) surface of the structure by the interface conditions. Quadratic conforming elements (with C^0 continuity) are used for the discretization of the surfaces of the elastic structure. For the verification of the coupled BIE formulations, spherical shells of different thickness (including nonuniform thickness) and materials, and a cylindrical shell structure, are tested for radiation and scattering problems. Very satisfactory results are obtained, which clearly demonstrate the effectiveness and accuracy of the developed BEM/BEM approach to the structural acoustic problems for thin shell-like structures. Efforts are underway to further improve the computational efficiency of the developed BEM/BEM approach and to study the multidomain BEM for slender structures. Results will be reported in subsequent papers.

I. BOUNDARY INTEGRAL EQUATION FORMULATION

Consider a 3D elastic thin structure (V) of an arbitrary shape and immersed in an acoustic media (E) with its outer surface denoted by S_a and inner surface by S_b (Fig. 1). The normal on either surface is defined as pointing away from the elastic domain. We consider only time-harmonic wave motion. The acoustic field is assumed to be inviscid and the elastic structure is assumed to be homogeneous, isotropic, and linearly elastic. Body forces are assumed to be negli-

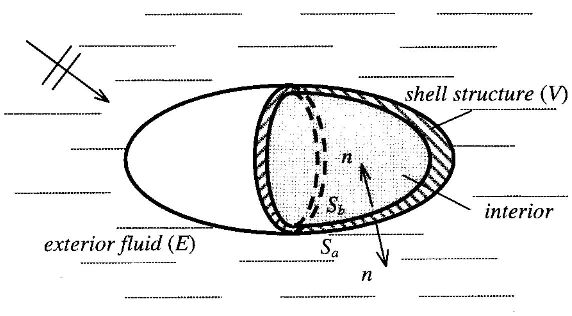

FIG. 1. A 3D shell structure immersed in fluid.

gible. Under these conditions, the wave equation governing the elastic domain (V) can be written as (index notation is used in this paper)

$$(c_1^2-c_2^2)u_{k,ki}(P)+c_2^2u_{i,kk}(P)+\omega^2u_i(P)=0,\ \ \forall P\in V, \tag{1}$$

in which c_1 and c_2 are the wave speeds of the pressure wave (P-wave) and shear wave (S-wave), respectively; u_i the displacement at a point $P\in V$; ω the angular frequency of oscillation. The dependence of u_i on ω has been suppressed. The governing equation for the acoustic domain is the Helmholtz equation,

$$\nabla^2\phi(P)+k^2\phi(P)=0,\ \ \forall P\in E, \tag{2}$$

where $\phi=\phi^S+\phi^I$ is the total disturbed acoustic pressure at a point P, ϕ^S the scattered wave, ϕ^I the incident wave for a scattering problem, $k=\omega/c$ the wave number, and c the speed of sound in the fluid.

On the two surfaces S_a and S_b of the structure, boundary or interface conditions need to be specified. On surface S_b, harmonic excitations in the form of surface displacement or surface traction can be applied, corresponding to a well-posed boundary value problem. On S_a, where the two domains are in contact, the following interface conditions are specified:

(a) The normal derivative of the acoustic pressure is related to the displacement in the normal direction as

$$\frac{\partial\phi}{\partial n}=\rho_f\omega^2u_n, \tag{3a}$$

where ρ_f is the mean density of the fluid, and u_n the normal component of the displacement.

(b) The normal stress is equal to the acoustic pressure such that

$$t_i=-\phi n_i, \tag{3b}$$

where t_i is the traction and n_i the components of the normal ($i=1,2,3$) in the global coordinates. In addition to these conditions, the acoustic pressure field must satisfy the Sommerfeld radiation condition at infinity, which is automatically satisfied by the BIE.

For the 3D elastodynamic problem, the integral representation of Eq. (1) may be written in the following form:

$$C_{ij}(P_0)u_j(P_0)=\int_S U_{ij}(P,P_0)t_j(P)dS(P) - \int_S T_{ij}(P,P_0)u_j(P)dS(P), \tag{4}$$

in which U_{ij} and T_{ij} are the dynamic displacement and traction kernels, respectively; P the field point and P_0 the source point, S the boundary of the elastic structure ($S=S_a\cup S_b$), and the coefficient $C_{ij}(P_0)=\delta_{ij}$, $1/2\delta_{ij}$, or 0 when the source point P_0 is in the interior region V, on the boundary S (if it is smooth) or in the exterior region E, respectively (δ_{ij} is the Kronecker delta). The second integral in Eq. (4), when P_0 is on the boundary S, is of the Cauchy principle value (CPV) type, which requires delicate numerical quadrature in general. This CPV integral can be avoided by recasting Eq. (4) into a weakly singular form[26] (with P_0 on S) as

$$\int_S[T_{ij}(P,P_0)-\overline{T}_{ij}(P,P_0)]u_j(P)dS(P)+\int_S\overline{T}_{ij}(P,P_0)\times[u_j(P)-u_j(P_0)]dS(P) =\int_S U_{ij}(P,P_0)t_j(P)dS(P),\ \ \forall P_0\in S,\ (S=S_a\cup S_b), \tag{5}$$

where $\overline{T}_{ij}$ is the static traction kernel. In Eq. (5) every integral is at most weakly singular and can be computed using the conventional quadrature.

For thin shell-like structures, the second integral in Eq. (5) becomes nearly singular when the source point is at one surface and the integration is performed on the nearby elements on the other surface. This nearly singular integral can be transformed into line integrals which are not singular at all.[29,30] With the help of these line integrals, Eq. (5) can be applied to shell-like structures and will not break down even when the thickness of the shell is very small.[29] Certainly, one can simply increase the number of integration points or use subdivisions on the element to deal with the nearly singular integrals in the BEM as applied for thin bodies. However, this approach has been found inefficient and prohibitively expensive for computing such integrals.[30]

For the acoustic domain embracing the elastic structure, the conventional boundary integral representation of Eq. (2) is the Helmholtz integral (note the direction of the normal n, Fig. 1),

$$C(P_0)\phi(P_0)=\int_{S_a}\left[\frac{\partial G(P,P_0)}{\partial n}\phi(P) - G(P,P_0)\frac{\partial\phi(P)}{\partial n}\right]dS(P)+\phi^I(P_0), \tag{6}$$

where $G(P,P_0)$ $(=e^{ikr}/4\pi r$, with $r=|\mathbf{P}_0\mathbf{P}|)$ is the full space Green's functions, and the coefficient $C(P_0)=1$, 1/2, or 0 when the source point P_0 is in E, on the boundary S_a (if it is smooth) or in V, respectively. When the source point P_0 is on the boundary S_a, the integral for both integrands is weakly singular, contrary to the case of Eq. (4). However, when the conventional BIE (6) is applied to exterior domains, nonunique solutions will arise at the frequencies corresponding to the eigenfrequencies of the interior domain. It was shown in Refs. 12–15 that with the use of the well-known Burton and Miller formulation, that is, a linear combination of CBIE and HBIE as shown symbolically by

$$\text{CBIE}+\beta\text{HBIE}=0 \quad (\beta=\text{constant}), \tag{7}$$

the fictitious eigenfrequency difficulty can be overcome effectively. It was also found in Refs. 18 and 19 that the thin-shape breakdown of the CBIE can be solved as well by using this composite BIE formulation.

The HBIE in Eq. (7) is readily obtained by taking the directional derivative of Eq. (6) in the direction n_0,

$$\frac{\partial\phi(P_0)}{\partial n_0}=\int_{S_a}\left[\frac{\partial^2 G(P,P_0)}{\partial n\partial n_0}\phi(P)-\frac{\partial G(P,P_0)}{\partial n_0}\frac{\partial\phi(P)}{\partial n}\right]dS(P)+\frac{\partial\phi^I(P_0)}{\partial n_0},\quad \forall P_0\in E. \tag{8}$$

Equation (8) can be written in the weakly singular form[15] (with the source point P_0 on the boundary S_a) as

$$\begin{aligned}&\frac{\partial\phi(P_0)}{\partial n_0}-\int_{S_a}\frac{\partial^2\bar{G}(P,P_0)}{\partial n\partial n_0}\left[\phi(P)-\phi(P_0)-\frac{\partial\phi(P_0)}{\partial\xi_\alpha}(\xi_\alpha-\xi_{0\alpha})\right]dS(P)-\int_{S_a}\frac{\partial^2}{\partial n\partial n_0}[G(P,P_0)-\bar{G}(P,P_0)]\phi(P)dS(P)\\&\quad -e_{\alpha k}\frac{\partial\phi(P_0)}{\partial\xi_\alpha}\int_{S_a}\left[\frac{\partial\bar{G}(P,P_0)}{\partial n_0}n_k(P)+\frac{\partial\bar{G}(P,P_0)}{\partial n}n_k(P_0)\right]dS(P)\\&\quad=-\int_{S_a}\left[\frac{\partial G(P,P_0)}{\partial n_0}+\frac{\partial\bar{G}(P,P_0)}{\partial n}\right]\frac{\partial\phi(P)}{\partial n}dS(P)+\int_{S_a}\frac{\partial\bar{G}(P,P_0)}{\partial n}\left[\frac{\partial\phi(P)}{\partial n}-\frac{\partial\phi(P_0)}{\partial n}\right]dS(P)+\frac{\partial\phi^I(P_0)}{\partial n_0},\quad\forall P_0\in S_a,\end{aligned} \tag{9}$$

where $\bar{G}$ is the static kernel, ξ_α and $\xi_{0\alpha}$ ($\alpha=1,2$) the two tangential coordinates of the points P and P_0, in a local coordinate $O\xi_1\xi_2\xi_3$ ($\xi_3=n$), respectively, and $e_{\alpha k}=\partial\xi_\alpha/\partial x_k$ ($k=1,2,3$) are the first two column vectors of the inverse of the Jacobian matrix.[15]

For the hypersingular integral in (8) to exist as the source point P_0 approaches the boundary or for the weakly singular forms in (9) to work, the density function $\phi(P)$ is required, in theory, to have continuous tangential derivatives ($C^{1,\alpha}$ continuity) in the neighborhood of the source point P_0. This smoothness requirement imposes severe limitations to the applications of HBIEs. For example, this smoothness requirement will exclude, theoretically, the use of C^0 boundary elements, such as the conforming quadratic elements, in the discretizations of HBIEs. Relaxation of this smoothness requirement for HBIEs has been attempted by several authors (see, e.g., Refs. 12–14, 37 and 38). The validation of this relaxation has also been provided in Refs. 15, 39, and 40. It has been postulated in Ref. 15 that the original $C^{1,\alpha}$ continuity requirement on the density function in the HBIE formulations can be relaxed to piecewise $C^{1,\alpha}$ continuity in the numerical implementation of the HBIEs as in the form of Eq. (9), so that conforming quadratic elements can be applied. Converged and very good numerical results have been obtained by adopting this strategy for the acoustic problems.[15] However, for domains with edges and corners, the use of conforming quadratic elements for Eq. (9) is not straightforward. Techniques, such as using coincident nodes, to deal with these situations, need to be tested.

II. DISCRETIZATION OF THE BIE

To obtain the numerical solution of Eqs. (5) and (7), surfaces S_a and S_b are discretized using isoparametric quadratic elements (Fig. 2). The discretized form of Eq. (5) can be expressed in matrix form as

$$\begin{bmatrix}\mathbf{U_{aa}} & \mathbf{U_{ab}}\\ \mathbf{U_{ba}} & \mathbf{U_{bb}}\end{bmatrix}\begin{Bmatrix}\mathbf{t_a}\\ \mathbf{t_b}\end{Bmatrix}+\begin{bmatrix}\mathbf{T_{aa}} & \mathbf{T_{ab}}\\ \mathbf{T_{ba}} & \mathbf{T_{bb}}\end{bmatrix}\begin{Bmatrix}\mathbf{u_a}\\ \mathbf{u_b}\end{Bmatrix}=\begin{Bmatrix}\mathbf{0}\\ \mathbf{0}\end{Bmatrix}, \tag{10}$$

in which subscripts a and b refer to the outer surface S_a and inner surface S_b, respectively; matrices $\mathbf{U}$ and $\mathbf{T}$ are from the displacement and traction kernels, respectively; $\mathbf{u}$ and $\mathbf{t}$ are the displacement and traction vectors, respectively. The total number of elements and nodes on S_a are denoted by M_a and N_a, respectively, and similarly M_b and N_b for the number of elements and nodes on S_b. The two coefficient matrices in Eq. (10) are square matrices of the dimension $3N$ by $3N(N=N_a+N_b)$. Since $\mathbf{t_a}$, $\mathbf{u_a}$, and $\mathbf{t_b}$ (or $\mathbf{u_b}$) are unknowns, additional information will be needed from the acoustic domain in order to solve the coupled structural acoustic problem.

The model for the acoustic field shares the same mesh with the model for the elastic field on the interface surface.

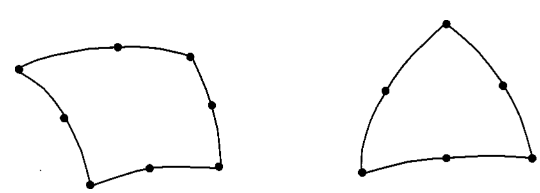

FIG. 2. Conforming quadratic boundary elements.

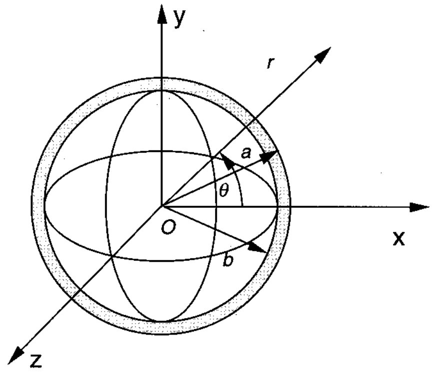

FIG. 3. A spherical shell with uniform thickness (outer radius$=a$, inner radius$=b$).

The linear system of equations obtained after discretization of the acoustic BIE (7) can be written in the following matrix form:

$$\mathbf{G}\frac{\partial \boldsymbol{\phi}}{\partial n}+\mathbf{H}\boldsymbol{\phi}=\boldsymbol{\phi}^{\mathbf{I}}, \tag{11}$$

where $\mathbf{G}$ and $\mathbf{H}$ are both N_a by N_a matrices and $\boldsymbol{\phi}^{\mathbf{I}}$ is the known vector from the incident wave. For a coupled structural acoustic problem, both $(\partial\boldsymbol{\phi}/\partial n)$ and $\boldsymbol{\phi}$ on the surface (S_a) are unknowns but are related to the unknowns of the elastic domain ($\mathbf{t_a}$ and $\mathbf{u_a}$) through the interface conditions [Eqs. (3a) and (3b)]. By using the interface conditions, two sets of these unknowns can be eliminated and the resulting linear system of equations can be expressed in matrix form as $(\partial\boldsymbol{\phi}/\partial n)$ and $\mathbf{t_a}$ are eliminated, $\mathbf{t_b}$ is assumed known),

$$\begin{bmatrix} \mathbf{H} & \mathbf{D} & \mathbf{0} \\ \mathbf{E_a} & \mathbf{T_{aa}} & \mathbf{T_{ab}} \\ \mathbf{E_b} & \mathbf{T_{ba}} & \mathbf{T_{bb}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\phi} \\ \mathbf{u_a} \\ \mathbf{u_b} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\phi}^{1} \\ -\mathbf{U_{ab}t_b} \\ -\mathbf{U_{bb}t_b} \end{Bmatrix}, \tag{12}$$

in which $\mathbf{D}=\rho_f\omega^2\mathbf{G}\nu$ is an N by $3N$ matrix and $\boldsymbol{\nu}$ is an N by $3N$ matrix in the following form:

$$\boldsymbol{\nu}=\begin{bmatrix} \mathbf{n}_1^T & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{n}_2^T & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{n}_N^T \end{bmatrix}, \tag{13}$$

where $\mathbf{n}_\alpha^T$ is the surface normal vector at node α ($\alpha=1,2,...,N$). Also in Eq. (12), $\mathbf{E_a}=-\mathbf{U_{aa}}\boldsymbol{\nu}^T$ and $\mathbf{E_b}=-\mathbf{U_{ab}}\boldsymbol{\nu}^T$ are matrices of the dimension $3N$ by N.

III. NUMERICAL EXAMPLES

First, the developed BEM/BEM approach was tested on radiation and scattering problems using a spherical shell (Fig. 3) with outer radius $a=1$ and thickness $h=0.5$, 0.05, and 0.01 m, respectively. Quadratic elements were used (Fig. 2) on both S_a and S_b. Four BEM meshes with increasing total numbers of elements (64, 112, 160, and 306) were used. Three typical materials, steel (Young's modulus $E=2.07\times10^{11}$ Pa, Poisson's ratio $\nu=0.3$, and density $\rho=7810\,\mathrm{kg/m^3}$), aluminum ($E=7.10\times10^{10}$ Pa, $\nu=0.33$, $\rho=2700\,\mathrm{kg/m^3}$), and hard rubber ($E=2.30\times10^{9}$ Pa, $\nu=0.4$, $\rho=2117\,\mathrm{kg/m^3}$), were used for the shell structures. The surrounding acoustic media were assumed to be seawater in all the cases; the density and speed of sound used are 1026 $\mathrm{kg/m^3}$ and 1500 m/s, respectively.

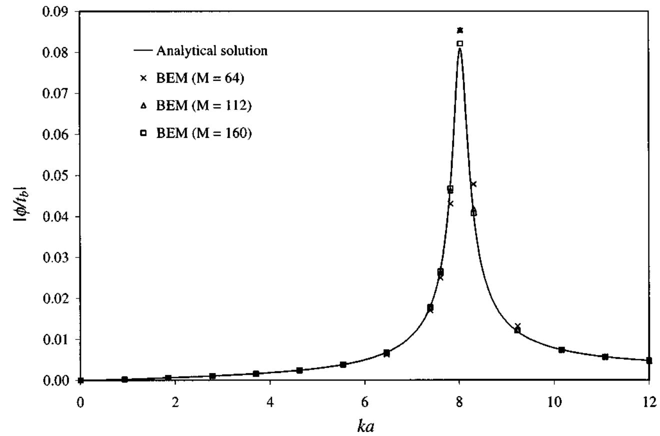

FIG. 4. Normalized radiated sound pressure from a steel spherical shell ($r=5a$, $h/a=0.5$).

For radiation problems, uniform time-harmonic pressure was applied on S_b with amplitude $t_b=1\times10^6$ N/m and the angular frequency being given. The radiated wave by a steel spherical shell with the thickness to radius ratio $h/a=0.5$ was studied first. The normalized sound pressure calculated at a distance $r=5a$ from the center of the shell is plotted in Fig. 4 versus ka for three different meshes. Very fast convergence of the BEM solution is observed as compared to the analytical solution (given in Ref. 36 where typographical errors have been corrected), although the convergence at the resonant frequency is slower than those at other frequencies. Since the mesh with $M=112$ already gives very good results except at the resonant frequency (Fig. 4), we used this mesh for the next two test cases on radiation problem. Figure 5 shows the BEM solution of the normalized radiated sound pressure plotted versus ka from a thin spherical shell ($h/a=0.01$) for three different materials. The BEM solutions match the analytical solution as expected. It is noted that the resonance occurs at lower frequency when the material of the shell is softer. The effect of the thickness of a steel spherical shell on the radiated sound field is shown in Fig. 6, where the

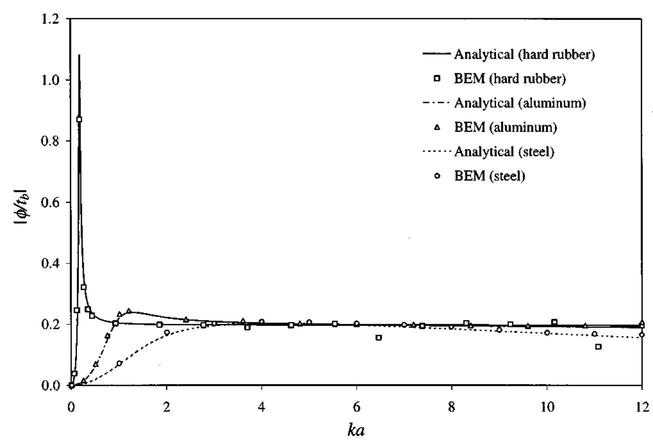

FIG. 5. Normalized radiated sound pressure from a thin spherical shell of different material ($r=5a$, $h/a=0.01$, $M=112$).

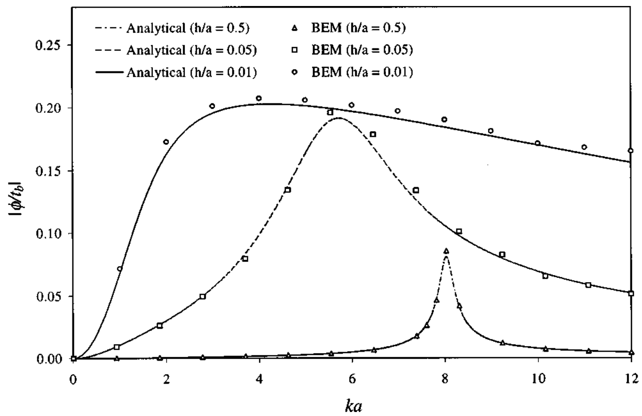

FIG. 6. Normalized radiated sound pressure from steel spherical shells with different thickness ($r=5a$, $M=112$).

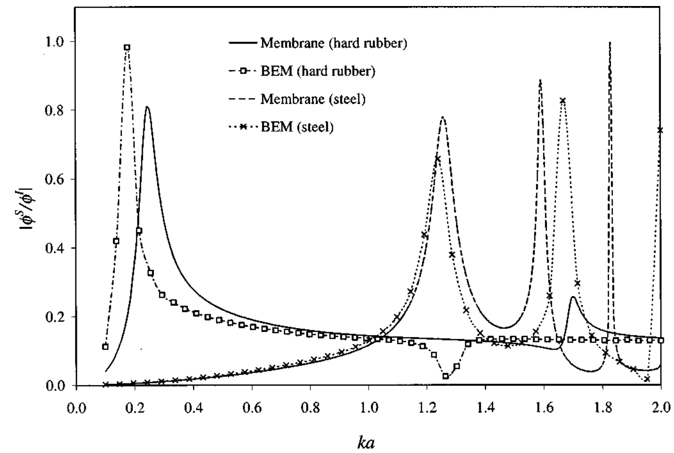

FIG. 8. Normalized backscattered sound pressure by a thin spherical shell of different materials ($r=5a$, $h/a=0.01$, $M=112$).

normalized radiated sound pressure at $r=5a$ is plotted versus *ka*.

For scattering problems, an elastic spherical shell impinged upon by an incident wave ϕ^I traveling in the positive *x* direction was considered. The inner surface of the spherical shell is assumed traction-free. The BEM solution using the mesh with $M=112$ is found to be a good approximation. It is therefore used in the test cases for scattering problem. First, the effect of thickness on the backscattering by the spherical shell is studied. A steel spherical shell with $h/a=0.05$ and $h/a=0.01$ was tested. Figure 7 shows the BEM solutions of the normalized backscattered sound pressure at $r=5a$ plotted versus *ka*. The membrane solutions[2] are also shown in the figure for comparison. The membrane model assumes that flexural stresses are negligible as compared to membrane stresses. This means that the membrane solution is a good approximation only when the shell is thin enough and the frequency is low. It can be seen that the BEM solution agrees with the membrane solution at low frequencies even when resonant frequencies are involved. At higher frequencies, large deviation can be observed near the resonant frequencies. Since the BEM solution is based on rigorous 3D elastodynamics, it is considered more accurate than the membrane solution.

The effect of different composition of the shell was studied next using a hard rubber spherical shell and a steel spherical shell with $h/a=0.01$. Figure 8 shows the normalized backscattered sound pressure at $r=5a$ plotted versus *ka*. It can be seen that the first resonant frequency for the hard rubber shell occurs at a lower frequency than that of the steel shell.

A test case on spherical shell with nonuniform thickness (Fig. 9) was performed with no additional modeling effort involved. With its outer surface and inner surface forming a sphere and a spheroid, respectively, the thickness of the spherical shell varies. The three axes of the spheroid were denoted by b_x, b_y, and b_z, respectively (Fig. 9). The radiated wave from a steel shell with $b_x=0.95a$ and $b_y=b_z=0.99a$ was calculated with uniform pressure t_b applied on the inner surface. Figure 10 shows the normalized radiated pressure at $r=5a$ in the direction of $\theta=0$ and 90 deg at frequencies up to $ka=12$. Two meshes with $M=112$ and $M=160$ are used in the calculation. The result calculated with 112 elements is shown since the mesh with $M=160$ provided only little improvement. The results from uniform thickness spherical shell with $h/a=0.05$ and 0.01 are shown in the same figure for comparison. It can be seen that the result for the nonuniform thickness shell in both directions approximates the average of the results from the two uniform thickness shells at low frequencies ($ka<3$). At higher frequencies, however, significant differences between the results for the two directions can be observed.

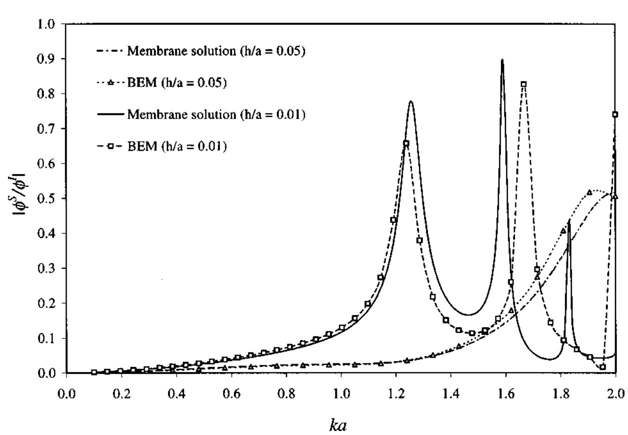

FIG. 7. Normalized backscattered sound pressure by a steel spherical shell with varying thickness ($r=5a$, $M=112$).

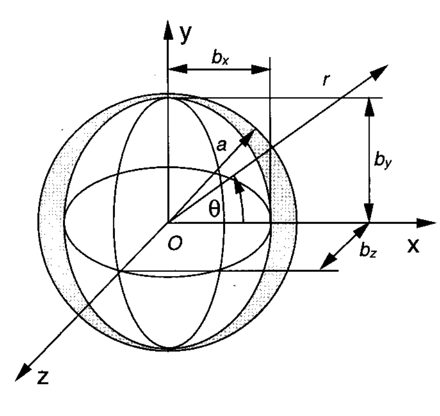

FIG. 9. A spherical shell with nonuniform thickness.

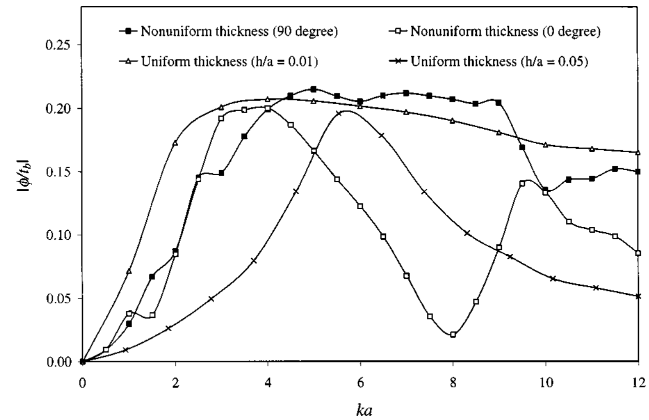

FIG. 10. Normalized radiated sound pressure from a steel spherical shell with nonuniform thickness ($r=5a$, $M=112$, $b_x=0.95a$, $b_y=b_z=0.99a$).

Finally, scattering from a cylindrical (capsulelike) thin shell (Fig. 11) made of steel is studied with a total of 216 elements (626 nodes). The incident wave is from the x-direction and the sound pressures from backscattering (at $x=-10$ m, $y=z=0$) and forward scattering (at $x=10$ m, $y=z=0$) from the elastic shell are plotted in Fig. 12. For comparisons, the results from the acoustics BEM[15] (assuming the shell to be rigid) and the current coupled BEM with large Young's modulus and density for the shell are also plotted in the figure. These latter two results agree very well for the rigid case, as expected. For forward scattering, the result for steel shell using the coupled BEM oscillates around the data for the rigid shell case. However, the result for backscattering from the steel shell using the coupled BEM differs significantly from the corresponding rigid shell case, especially when the frequencies are higher. This further signifies the necessity of a coupled analysis for structural acoustic problems.

IV. DISCUSSIONS

A unified BEM/BEM approach to sound–structure interaction problems in the frequency domain for shell-like structures is developed in this paper. The formulation is valid for general loading conditions and for all frequencies. The Burton and Miller composite BIE formulation is employed to overcome the fictitious eigenfrequency difficulty and the thin-shape breakdown for the acoustic domain. The hypersingular integrals involved are transformed into the weakly singular forms and evaluated by ordinary quadrature. The nearly singular integrals existing in the 3D elastodynamic BIE for thin-shell-like structures are treated by semianalytical methods and evaluated accurately. Numerical examples on radiation and scattering problems from bulky and thin spherical and cylindrical shells (including ones with nonuniform thickness) clearly demonstrate the effectiveness and accuracy of the developed approach.

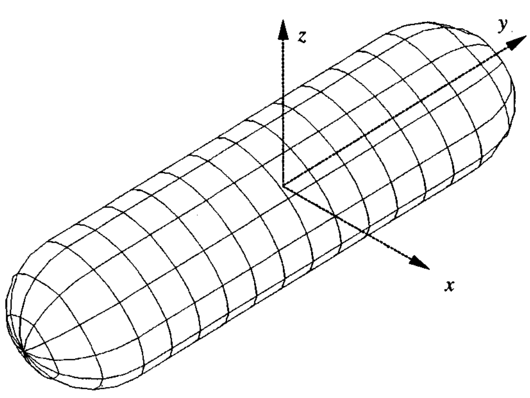

FIG. 11. A cylindrical (capsulelike) shell structure with thickness = 0.01, radius = 1.0, and total length = 7.0 m.

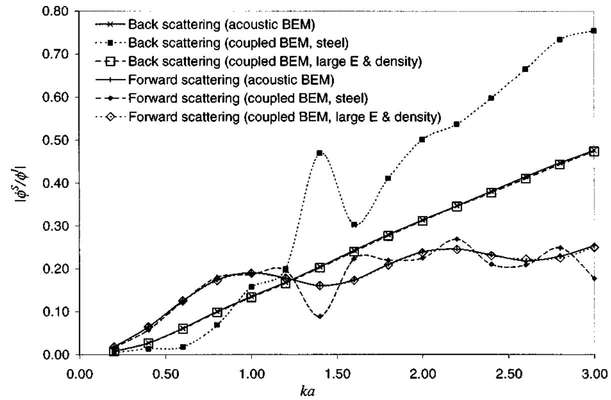

FIG. 12. Scattered sound pressure from the capsulelike shell structure.

There are many advantages in the developed BEM/BEM approach to the structural acoustic analysis. First of all, this approach renders high accuracy for both acoustic and elastic domains due to the semianalytical nature of the BEM. Second, the coupling effect is modeled effectively and efficiently by sharing the same surface mesh on the interface between the elastic domain and the acoustic domain. Moreover, the unified BEM approach is much easier in modeling than the other methods for structures with complicated features in either the interior (e.g., stiffeners) or the exterior (e.g., rudders, turbo blades of a submarine). Multiple scatterers (e.g., an array of shell structures) can also be modeled readily by the BEM with the multidomain technique. Finally, shell structures with nonuniform thickness or coatings (layered shell structures) can be handled accurately by the BIE formulations. The remaining major issue is the computational efficiency, as the BEM usually generates fully populated matrices, although of smaller size than those of the matrices from the FEM/IEM approach. With the recent development of iterative solvers for asymmetric and dense complex systems (see, e.g., Ref. 41), which can dramatically increase the speed of solving large linear systems, this efficiency concern may be eased in the near future.

Studies on efficient solution techniques, including iterative solvers, and more complicated shell structures using the developed BEM/BEM approach to structural acoustics are underway and the results will be reported in subsequent papers. Multiple scatterers and layered shell structures (shells with coatings) will be interesting and challenging future research topics.

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