An Improved Solution Strategy for the Composite BIE for 3-D Acoustic Problems

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Summary

This paper addresses mainly the computational efficiency issues in applying the composite boundary integral equation (BIE) formulation (a linear combination of the conventional and hypersingular BIEs) for 3-D exterior acoustic wave problems. The considered issues are: (1) the relaxation of the smoothness requirement imposed by theory to the hypersingular BIE, and hence the use of C^0 boundary elements to the composite BIE based on this relaxation; (2) the use of iterative solvers to speedup the solution time in solving the BIEs; (3) a mechanism to invoke the hypersingular BIE in the composite BIE formulation only when it is needed, i.e., only when the conventional BIE fails, so that the formation time used for the composite BIE can be reduced to a minimum. Numerical examples of both scattering and radiation problems show that the conforming quadratic elements, which are C^0 continuous, can be applied to the composite BIE to provide efficient, accurate, and reliable results; and the iterative solver can cut the solution time by more than an order for problems of larger size. Study on the technique in reducing the formation time in the composite BIE applications is underway and results will be reported at the ICES'98/IABEM conference.

The Composite BIE Formulation and Related Issues

The conventional boundary integral equation (CBIE) is given by:

$$C(P_o)f(P_o)=\int_S\left[G(P,P_o)\frac{\partial f(P)}{\partial n}-\frac{\partial G(P,P_o)}{\partial n}f(P)\right]dS(P)+f^I(P_o),\quad P_o\in S \tag{1}$$

where f is the total acoustic wave satisfying the Helmholtz equation for time harmonic waves, f^I is a incident wave, $G(P,P_o)=e^{ikr}/(4\pi r)$ is the Green's function, and $C(P_o)$ = 1/2 if the boundary S is smooth. Equation (1) is a singular form of the CBIE which can be converted into a weakly-singular form readily. The hypersingular BIE (HBIE), which is the derivative of the CBIE (1), has many variations or forms which may contain hypersingular, singular or weakly-singular integrals. One new weakly-singular form of the hypersingular BIE, given below, was derived recently in [1]:

$$\begin{aligned}
&\frac{\partial f(P_o)}{\partial n_o}+\int_S\frac{\partial^2\overline{G}(P,P_o)}{\partial n\partial n_o}\left[f(P)-f(P_o)-\frac{\partial f(P_o)}{\partial x_\alpha}(x_\alpha-x_{o\alpha})\right]dS(P)\\
&+\int_S\frac{\partial^2}{\partial n\partial n_o}\left[G(P,P_o)-\overline{G}(P,P_o)\right]f(p)dS(P)\\
&+e_{\alpha k}\frac{\partial f(P_o)}{\partial x_\alpha}\int_S\left[\frac{\partial\overline{G}(P,P_o)}{\partial n_o}n_k(P)+\frac{\partial\overline{G}(P,P_o)}{\partial n}n_k(P_o)\right]dS(P)\\
&=\int_S\left[\frac{\partial G(P,P_o)}{\partial n_o}+\frac{\partial\overline{G}(P,P_o)}{\partial n}\right]\frac{\partial f(P)}{\partial n}dS(P)
\end{aligned}$$

$$-\int_S \frac{\partial \bar{G}(P,P_o)}{\partial n}\left[\frac{\partial f(P)}{\partial n}-\frac{\partial f(P_o)}{\partial n}\right]dS(P)+\frac{\partial f^I(P_o)}{\partial n_o}, \qquad \forall P_o \in S, \tag{2}$$

in which $\bar{G}=1/(4\pi r)$, and x_1 and x_2 are the first two (tangential) coordinates of a local curvilinear coordinate system $Ox_1x_2x_3$ with origin at the source point P_o. All the integrals in (2) are at most weakly-singular, *if* $f(P)$ *has continuous first derivatives*. This condition is consistent with the $C^{1,\alpha}$ continuity requirement on the density function $f(P)$ for HBIEs to be meaningful [2].

The composite BIE formulation, proposed by Burton and Miller [3] for exterior acoustic wave problems, uses a linear combination of the CBIE (1) and HBIE (2). The composite BIE has been demonstrated to be a mathematically-sound approach which can not only overcome the fictitious eigenfrequency difficulty (FED), but also the thin-shape breakdown (TSB) problem [4, 5], both existing in the conventional BIE. This composite BIE has been used successfully for acoustic wave problems, see, e.g., [6-8]. However, wide applications of this composite BIE formulation are still limited, due to the seemingly difficulty in dealing with the hypersingular integrals present in this formulation and the associated smoothness issue.

It has been a dilemma for a long time in using HBIEs that on one hand, theory dictates that smoothness requirement [2] must be satisfied for the HBIEs to be meaningful; on the other hand, good numerical results have been obtained by using conforming C^0 boundary elements for various forms of the HBIEs [6-8]. This unsettled situation for the HBIEs may be one of the main reasons for their slow acceptance in the BEM community, even though the formulation based on the HBIE has been proved to be a very sound and effective approach for acoustic problems. In light of the recent thinking [9] on the smoothness requirement for HBIEs and its relaxation, and the continued successful studies using C^0 conforming elements for HBIEs (e.g., [6-8]), it is necessary to re-address the issue of smoothness and its relaxation for HBIEs, and clear the way for the applications of the HBIEs in acoustics. With careful examination of the weakly-singular form of the HBIE (2), it is postulated in [1] that the original $C^{1,\alpha}$ continuity requirement on the density function, can be relaxed to *piecewise* $C^{1,\alpha}$ continuity in the *numerical* implementation of the *weakly-singular forms* of the hypersingular BIE. This relaxation means that conforming linear, quadratic, and other higher-order elements, as well as nonconforming elements (including the constant elements), can be applied to the weakly-singular forms of the HBIEs. The important task in this area is then to provide the proof for the convergence of the weakly-singular forms of the HBIEs with conforming elements.

Numerical Examples

Numerical studies on the scattering and radiation from cylindrical and box-like bodies were conducted to verify the composite BIE with the C^0 conforming quadratic elements and the suggested solution strategies, including an iterative solver.

The first example is a cylindrical body (a pulsating capsule) as shown in Fig. 1. Since no analytical solutions are readily available for this problem, the commercial boundary element software *COMET/Acoustics* is employed in the verification for the radiation problem. The same mesh with 216 elements and 626 nodes (Fig. 1) is used for both *COMET/Acoustics* and the developed composite BIE code. A uniform velocity of unit magnitude is applied on the whole surface of the cylinder. Radiated waves for frequencies from 0 to 250 Hz (with 100 frequency steps), at the two points (10, 0, 0) in the lateral direction and (0, 10, 0) in the axis direction, are plotted in Fig. 2 and Fig. 3, respectively. *COMET/Acoustics* direct

BIE is based on the same conventional BIE formulation as the one used in this study, and suffers from the same fictitious eigenfrequency difficulty. The CHIEF method [10] is used in *COMET/Acoustics* to overcome this difficulty. Fig. 2 and Fig. 3 show that both the results using *COMET* direct BIE and the CBIE deteriorate near the four fictitious eigenfrequencies (134, 153, 185 and 224 Hz), while the results using *COMET* CHIEF and the composite BIE stay closely along a smooth curve, as expected.

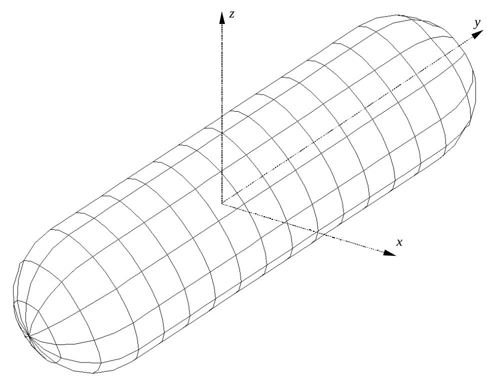

Figure 1. A capsule-like body with radius = 1.0m and total length = 7.0m.

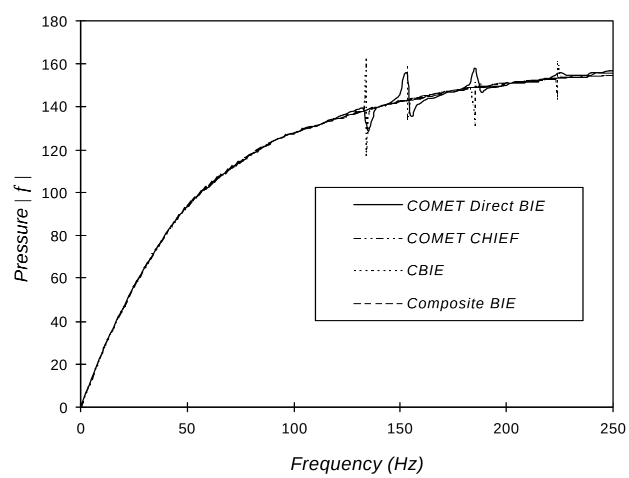

Figure 2. Radiated wave from the cylinder in the lateral direction.

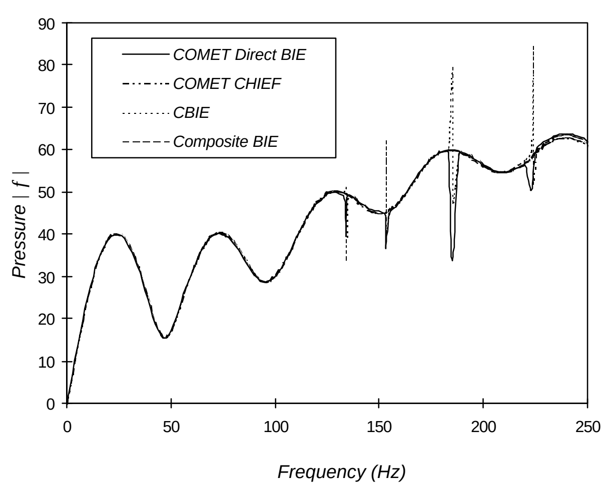

Figure 3. Radiated wave from the cylinder in the main axis direction.

The second example is a box-like structure, Fig. 4, which is used in [11]. The mesh has 198 elements and 596 nodes. The first five fictitious eigenfrequencies for this box using the direct CBIE formulation are 2449.5, 2520.5, 2634.6, 2786.5 and 2970.5 Hz by the formula given in [10]. It should be noted that this box geometry with edges and corners presents new problems for the HBIE formulation given in Eq. (2) which can not be applied, theoretically, to domains with edges or corners. This example will thus test the tolerance of the current HBIE to such problems, where unique or averaged normal is assumed at each node in the formulation and its discretization. The scattering problem is considered first for this box, with the incident wave coming in the *x*-direction (Fig. 4) and $\partial \phi / \partial n = 0$ on *S*. Scattered waves at the point (0, 0.75, 0) are plotted in Fig. 5. Again, results using the CBIE start to oscillate near the five fictitious eigenfrequencies. The condition numbers are also much higher near these frequencies. Results from the composite BIE follows a smooth curve, but have some noticeable discrepancies with the CBIE results away from the fictitious frequencies. This may be due to the corner problem with the current HBIE formulation. For the radiation problem, a uniform normal velocity of unit magnitude is assumed at one end (y = -0.25) of the box. The radiated pressure wave at the point (0, 0.75, 0) is plotted in Fig 6. Same conclusions can be drawn from this plot as from the previous two plots. Again, the difference between the results from the CBIE and composite BIE away from the fictitious eigenfrequencies may be caused by the presence of corners and edges in the model. The corner problem for the HBIE with conforming elements is being investigated in order to improve the accuracy of the composite BIE for such applications.

Finally, the preliminary test result on an iterative solver that can be used to solve general, complex linear systems of equations is shown in Fig. 7. The solver tested is CUCPL package which employs the coupled two-term QMR with look-ahead technique [12]. The test was done on a Pentium Pro 200 MHz PC with 64 Mb memory and under Windows NT. The result shows a dramatic improvement in the solution time using the iterative solver which is 52 times faster than the direct solver when DOF = 2562. For problems of this size, the ratio of the solution time to formation time (used to setup the linear system of equations) is about 4.7 for the direct solver and is only 0.089 for the iterative solver. This trend will continue with problems of even larger size, and hence reducing the formation time in the boundary element method will become a crucial issue in order to further improve its efficiency.

Based on the observation that the CBIE fails only at a limited number of frequencies for a frequency sweep calculation, the HBIE can be called upon only in the neighborhood of these fictitious frequencies, in order to reduce the formation time for the coefficient matrices. Schemes to monitor the quality of the CBIE is being established and tested. Results will be reported at the conference.

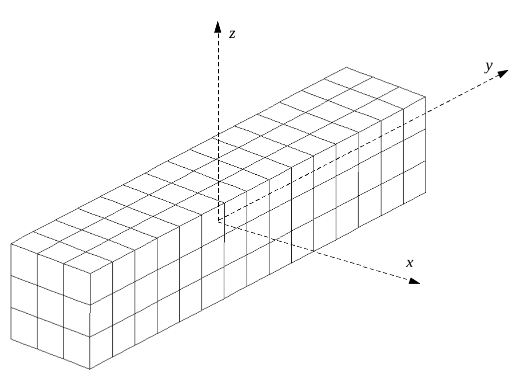

Figure 4. A box with length = 0.5m, width = 0.1m and height = 0.1m.

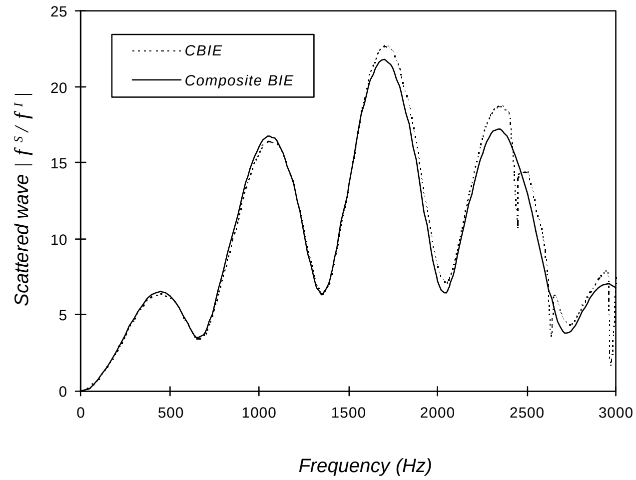

Figure 5. Scattering from the box in the main axis direction, incident wave in the lateral direction.

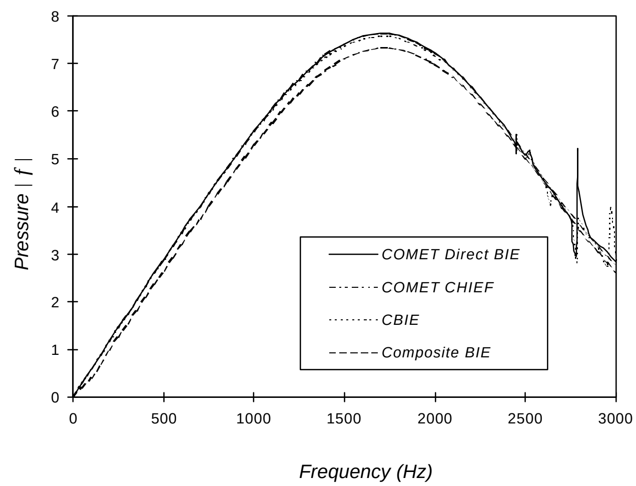

Figure 6. Radiated wave from the box in the main axis direction.

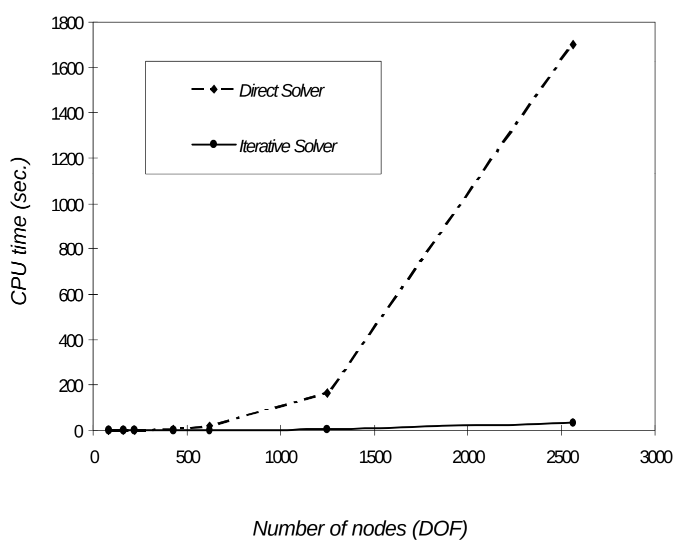

Figure 7. Solution time for a sphere model (CBIE; convergence tolerance ε = 5.0E-6).

Conclusion

Several issues related to improving the computational efficiency of the composite BIE have been investigated. Numerical examples, ranging from scattering and radiation problems with different geometries, clearly demonstrate the effectiveness and efficiency of the improved composite BIE approach to 3-D acoustic problems. More numerical examples will be presented at the ICES'98/IABEM conference.

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