Green-function method in hydrodynamics

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- **Fluid flows typically are appreciably affected by viscosity (especially internal flows)**
- **At least two notable exceptions :**
  - **(i) Flows that are rapidly accelerated from rest**
  - **(ii) Waves, notably water waves**

Basic problems

Interactions between water waves and floating bodies :
mostly ships and offshore structures

- **Diffraction-radiation of waves by offshore structures**
- **Wavemaking by a ship steadily advancing in calm water**
- **Diffraction-radiation of waves by advancing ships**
  (ship motions in waves)

- **Most problems consider time-harmonic (regular) waves**
- **But unsteady motions are also important**
  (e.g. ship maneuvering in waves)

- **Most problems consider deep water**
- **Finite water depth is also important**
- **Effects of horizontal confinement (e.g. canal)**

- **Most problems consider monohull ships**
- **But multihull ships (catamarans) are also common**
- **Many other types of ships (ACV, SES, ...)**

Generic mathematical problem

Potential flow theory : $\mathbf{u} = \nabla\phi$

- **Laplace equation: $\nabla^2\phi = 0$ in flow region**
- **Neumann BC at body surface: $\nabla\phi \cdot \mathbf{n}$ given**
  at surface of rigid body (ship or offshore structure)
- **A farfield boundary condition**
- **A radiation condition ?** $\phi(\mathbf{x})\, e^{i(f - i\epsilon)t}$
- **Several types of free-surface boundary condition; e.g.**

$$\phi_z - f^2\phi - i\,2\tau\,\phi_x + F^2\phi_{xx} + 2\,i\,\epsilon(f\phi + iF\phi_x) = 0 \text{ at } z = 0$$

Main source of difficulties: free-surface boundary condition and related Green functions

Very brief history

Goes back a long way

- **Arguably to Michell's theory of ship wavemaking (1898)**
- **1970s : Formulation of alternative integral equations**
- **From 1970s until now : search for simplified Green functions**
- **1980s & 1990s : methods based on basic Green function $1/r$**
- **Unresolved issues & ongoing work, notably for ship motions**
- **More recently: research activities divided between CFD and Green-function methods, which are less popular but remain very important for practical purposes**

Some basic nontrivial issues

FUNDAMENTAL ISSUES :

• Basic questions about consistency of free-surface and rigid-body boundary conditions at intersection curve (ship waterline)

• Basic questions about line integral around ship waterline

• Are the classical linear boundary-value problems correct ?

• Unrealistic short gravity waves need to be filtered (not trivial) or effects of viscosity and surface-tension must be considered

TECHNICAL ISSUES :

• Dispersion relation for time-harmonic ship waves has several roots, except for offshore structures (no forward speed)

• These multiple roots correspond to several distinct systems of waves, with widely different wavelengths, that travel in various directions

• Green functions that satisfy the boundary condition at the free surface given by complicated singular double Fourier integral

• Complicated nearfield singularity of Green function

• Practical and reliable numerical calculations remain illusive

Some ongoing research

TECHNICAL ISSUES :

• How complicated do Green functions need to be? in fact, highly simplified Green functions have recently been obtained

• Basic numerical task can also be greatly simplified

• Practical and reliable numerical calculations are now feasible (CPU=10s for hull form approximated by 10,000 panels)

• Method only involves ORDINARY CONTINUOUS functions (cos , sin , exp) of REAL arguments

FUNDAMENTAL ISSUES :

• Classical boundary-value problem for flow about a ship steadily advancing in calm water is NOT consistent

• Preliminary results based on proposed alternative problem very encouraging

• This alternative boundary-value problem is now being further investigated

• New ideas for large-amplitude motions of a ship advancing through regular waves (frequency-domain study of ship motions)

Practical relevance and applications

Practical methods suited for routine applications are essential for design, notably at early stages (concept and preliminary design) that often involve a very large number of alternative designs, for systematic parametric studies, and for hydrodynamic hull-form optimization

The future?

This critical practical need
and significant recent theoretical developments
mean a very active and bright future, in my opinion